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The Effect of Edge Ratio and Fiber Orientation on Free Vibration Analysis of Laminated Composite Plates on Elastic Foundation

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Abstract

This study presents the effect of edge ratio and fiber orientation on free vibration analysis of simply supported antisymmetric thin and thick laminated composite plates (LCP) on elastic foundation. In the analysis, the foundation is modeled as two parameters Pasternak and Winkler type foundation. The equation of motion for laminated rectangular plates resting on elastic foundation is obtained through Hamilton’s principle. The closed form solutions are obtained by using Navier technique, and then fundamental frequencies are found by solving the results of eigenvalue problems. The numerical results obtained through the present analysis are presented, and compared with the previous studies in the literature.

Keywords: Laminated composite, Free vibration, Elastic foundation, Shear deformation plate theory

Elastik Zemin Üzerine Oturan Tabakalı Kompozit Plaklarda Kenar Oranlarının ve Fiber Açılarının Değişiminin Serbest Titreşim Analizi Üzerine Etkisi

Öz

Bu çalışmada, elastik zemin üzerine oturan basit mesnetli antisimetrik dizilimli tabakalı kompozit ince ve kalın plakların (LCP) plak kenar uzunluklarının oranının ve fiber yönelimlerinin, serbest titreşim analizi üzerine etkisi sunulmaktadır. Bu analizlerde, zemin Pasternak ve Winkler tipi iki zemin parametresi ile modellenmiştir. Hamilton prensipleri ile elastik zemin üzerindeki tabakalı kompozit dikdörtgen plakların hareket denklemleri elde edilmiştir. Navier tekniği kullanılarak kapalı form çözümleri elde edilmiş ve sonra özdeğer problemi çözülerek temel frekanslar bulunmuştur. Analizler ile elde edilen nümerik sonuçlar çalışmada sunulmuş ve literatürdeki çalışmalarla karşılaştırılmıştır.

Anahtar Kelimeler: Tabakalı kompozit, Serbest titreşim, Elastik zemin, Kayma deformasyon plak teorisi

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1. INTRODUCTION

Recently, due to the many paramount properties advanced composite materials such as laminated plates are found an application area in the engineering projects. Tremendous researches have been performed on the LCP to clarify the advantages of using these types of materials. One of the focused topics in research subject is the free vibration analysis of composite plates on elastic foundation.

Although LCP are of interest to many researchers, there is a dearth of study for angle-ply LCP in the literature. In this paper, both cross-ply and angle-ply LCP are analyzed. The aim of this paper is to present effecting of all plate parameters and Winkler-Pasternak soil parameters on free vibrations of LCP on elastic foundation.

In this research, free vibration analyses of anti-symmetrically LCP on elastic foundation are investigated in detail depend on the Winkler and Pasternak soil parameters, number of layers, plate thickness ratio, plate edge ratio and fiber angle orientation.

2. PREVIOUS STUDIES

Reissner theory [1] is one of the theories which include the shear deformation effect and many researchers have studied on the dynamic analysis of LCP by using Reissner theory. Nelson and Lorch [2] developed high order plate theories to appraise the shear strain of the LCP. Noor [3] has been examined the stability and vibration analysis of the composite plates. Reddy [4] and Qatu [5] used energy function to develop governing equations of LCP. Applying different plate theories, Reddy and Khdeir [6] investigated buckling and vibration analysis of LCP. Matsunaga [7], Kant and Swaminathan [8] have studied on the free vibrations of laminated thick plates using higher-order plate theory. Hui-Shen et al. [9] investigated dynamic behaviour of LCP on elastic foundation under thermomechanical loading. Many studies have been performed on characteristics of plates by Qatu [10]. Reddy [11] presented studies including the effect of shear deformation for composite plates. Dogan and Arslan [12] investigated the effect of dimension on mode-shapes of composite shells. Akavci et al. [13] examined dynamic behavior of LCP on elastic foundation by using First-order Shear Deformation Theory (FSDT).

3. MATERIALS AND METHODS

A lamina is produced with the isotropic homogenous fibers and matrix materials. Any point on a fiber, and/or on matrix and/or on matrix-fiber interface has crucial effect on the stiffness of the lamina. Due to the big variation on the properties of lamina from point to point, macro-mechanical properties of lamina are determined based on the statistical approach.

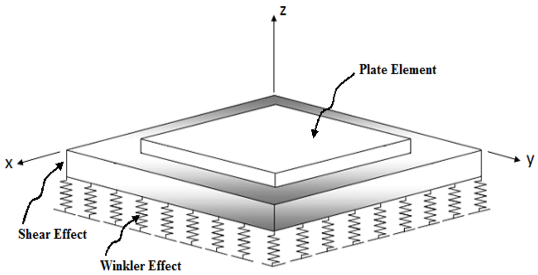

Figure 1. Laminated composite plate on elastic foundation

According to FSDT, the transverse normal do not cease perpendicular to the mid-surface after deformation. It will be assumed that the deformation of the plates is completely determined by the displacement of its middle surface. Using the given equation below (Eq.1) nth layer lamina plate stress-strain relationship can be defined in lamina coordinates,

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{xz}\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}\overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{13} & 0 & 0 & \overline{Q}_{16}\\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{23} & 0 & 0 & \overline{Q}_{26}\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{33} & 0 & 0 & \overline{Q}_{36}\\ 0 & 0 & 0 & \overline{Q}_{44} & \overline{Q}_{45} & 0\\ 0 & 0 & 0 & \overline{Q}_{45} & \overline{Q}_{55} & 0\\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{36} & 0 & 0 & \overline{Q}_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{xz}\\ \gamma_{xy}\end{bmatrix} \quad (1)$$

The displacement based on plate theory can be written as

$$u(x,y,z)=u_0(x,y)+z\,\varphi_x\,(x,y)$$
$$v(x,y,z)=v_0(x,y)+z\varphi_y\,(x,y)$$
$$w(x,y,z)=w_0(x,y) \qquad (2)$$

where u, v, w, φ_x and φ_y are displacements and rotations in x, y, z direction, orderly. u_o, v_o and w_o are mid-plane displacements.

Equation of motion for plate structures can be derived by Hamilton's principle

$$\delta\int_{t_1}^{t_2}(T+W-(U+U_F)dt=0 \qquad (3)$$

where T is the kinetic energy of the structure

$$T=\frac{\rho}{2}\int\left\{\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial v}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right\}dxdydz \qquad (4)$$

W is the work of the external forces

$$W=\int_x\int_y(q_xu+q_yv+q_zw+m_x\psi_x+m_y\,\psi_y)dxdy \qquad (5)$$

in which q_x, q_y, q_z, m_x,m_y are the external forces and moments, respectively. U is the strain energy and U_F is the spring strain energy defined as,

$$U=\frac{1}{2}\int\big(\sigma_x\varepsilon_x+\sigma_y\varepsilon_y+\sigma_z\varepsilon_z+\tau_{xy}\gamma_{xy}+\tau_{xz}\gamma_{xz}+\tau_{yz}\gamma_{yz}\big)dxdydz \qquad (6)$$

$$U_F=\frac{1}{2}\int\left(k_0w^2+k_1\left[\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]\right)dxdy \qquad (7)$$

Solving equation 3 gives set of equations called equations of motion for plate structures. This gives equation 8 in simplified form as,

$$\frac{\partial}{\partial x}N_x+\frac{\partial}{\partial y}N_{yx}+q_x=\bar{I}_1\ddot{u}^2+\bar{I}_2\ddot{\psi}_x^2$$
$$\frac{\partial}{\partial y}N_y+\frac{\partial}{\partial x}N_{xy}+q_y=\bar{I}_1\ddot{v}^2+\bar{I}_2\ddot{\psi}_y^2$$
$$\frac{\partial}{\partial x}Q_x+\frac{\partial}{\partial y}Q_y+q_z+k_0w+k_1\Delta^2w=\bar{I}_1\ddot{w}^2$$
$$\frac{\partial}{\partial x}M_x+\frac{\partial}{\partial y}M_{yx}-Q_x+m_x=\bar{I}_2\ddot{u}^2+\bar{I}_3\ddot{\psi}_x^2$$
$$\frac{\partial}{\partial y}M_y+\frac{\partial}{\partial x}M_{xy}-Q_y+m_y=\bar{I}_2\ddot{v}^2+\bar{I}_3\ddot{\psi}_y^2 \qquad (8)$$

Equation 8 is defined as equation of motion for thick shallow shell. The force and moment resultants are

$$\begin{bmatrix}N_x\\N_y\\N_{xy}\\M_x\\M_y\\M_{xy}\end{bmatrix}=\begin{bmatrix}A_{11}&A_{12}&A_{16}&B_{11}&B_{12}&B_{16}\\A_{12}&A_{22}&A_{26}&B_{12}&B_{22}&B_{26}\\A_{16}&A_{26}&A_{66}&B_{16}&B_{26}&B_{66}\\B_{11}&B_{12}&B_{16}&D_{11}&D_{12}&D_{16}\\B_{12}&B_{22}&B_{26}&D_{12}&D_{22}&D_{26}\\B_{16}&B_{26}&B_{66}&D_{16}&D_{26}&D_{66}\end{bmatrix}\begin{bmatrix}\varepsilon_{0x}\\\varepsilon_{0y}\\\gamma_{0xy}\\\kappa_x\\\kappa_y\\\tau\end{bmatrix}$$

$$\begin{bmatrix}Q_x\\Q_y\end{bmatrix}=\begin{bmatrix}A_{55}&A_{45}\\A_{45}&A_{44}\end{bmatrix}\begin{bmatrix}\gamma_{0xz}\\\gamma_{0yz}\end{bmatrix} \qquad (9)$$

$$\{A_{ij},B_{ij},D_{ij}\}=\int_{-h/2}^{h/2}\{1,z,z^2\}\overline{Q}_{ij}dz \qquad i,j=1,2,6$$
$$\{A_{ij}\}=\int_{-h/2}^{h/2}\overline{Q}_{ij}dz \qquad i,j=4,5 \qquad (10)$$
$$[I_1,I_2,I_3,I_4,I_5]=\int_{-h/2}^{h/2}\rho(1,z,z^2,z^3,z^4)dz$$

The Navier type solution might be implemented to thick and thin plates. This type solution assumes that the displacement section of the plates can be denoted as sine and cosine trigonometric functions.

Assume a plate with shear diaphragm boundaries on all edges. For simply supported thick plates, boundary conditions can be arranged as follows:

$$N_x = w_0 = v_0 = M_x = \psi_y = 0 \qquad x = 0,a$$
$$N_y = w_0 = u_0 = M_y = \psi_x = 0 \qquad y = 0,b \tag{11}$$

The displacement functions of satisfied the boundary conditions apply;

$$u_0(x,y,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} U_{mn} Cos(x_m x) Sin(y_n y) Sin(\omega_{mn} t)$$

$$v_0(x,y,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} V_{mn} Sin(x_m x) Cos(y_n y) Sin(\omega_{mn} t)$$

$$w_0(x,y,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} W_{mn} Sin(x_m x) Sin(y_n y) Sin(\omega_{mn} t)$$

$$\psi_x(x,y,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \psi_{xmn} Cos(x_m x) Sin(y_n y) Sin(\omega_{mn} t)$$

$$\psi_y(x,y,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \psi_{ymn} Sin(x_m x) Cos(y_n y) Sin(\omega_{mn} t) \tag{12}$$

where $x_m = m\pi/a$, $y_n = n\pi/b$.
Substituting the above equations into the equation of motion in matrix form,

$$\begin{bmatrix} M_{11} & 0 & 0 & M_{14} & 0 \\ 0 & M_{22} & 0 & 0 & M_{25} \\ 0 & 0 & M_{33} & 0 & 0 \\ M_{41} & 0 & 0 & M_{44} & 0 \\ 0 & M_{52} & 0 & 0 & M_{55} \end{bmatrix} \begin{bmatrix} \ddot{U}_{mn} \\ \ddot{V}_{mn} \\ \ddot{W}_{mn} \\ \ddot{\psi}_{xmn} \\ \ddot{\psi}_{ymn} \end{bmatrix} +$$
$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} \end{bmatrix} \begin{bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \psi_{xmn} \\ \psi_{ymn} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{13}$$

Following equation can be used directly to find the natural frequencies of free vibrations. The number of terms that taken into account in the m and n cycle is one (i.e. m=1 and n=1). Where the elements different from zero of M_{ij} and K_{ij}

$$[K_{mn}]\{\Delta\} + (\omega_{mn})^2 [M_{mn}]\{\Delta\} = 0 \tag{14}$$

$$\begin{aligned}
&K_{11}=-A_{11}x_m^2-A_{16}x_m y_n-A_{66}y_n^2 \\
&K_{12}=K_{21}=-A_{16}x_m^2-(A_{12}+A_{66})x_m y_n-A_{26}y_n^2 \\
&K_{14}=K_{41}=-B_{11}x_m^2-2B_{16}x_m y_n-B_{66}y_n^2 \\
&K_{15}=K_{51}=-B_{16}x_m^2-(B_{12}B_{66})\, x_m y_n-B_{26}y_n^2 \\
&K_{22}=-A_{66}x_m^2-A_{26}x_m y_n-A_{22}y_n^2 \\
&K_{24}=K_{42}=-(B_{12}+B_{66})\, x_m y_n \\
&K_{25}=K_{52}=-B_{66}x_m^2-B_{22}y_n^2 \\
&K_{33}=-A_{55}x_m^2-2A_{45}x_m y_n-A_{44}y_n^2-k_0-k_1(x_m^2+y_n^2) \\
&K_{34}=K_{43}=-A_{55}x_m-A_{45}y_n \\
&K_{35}=K_{53}=-A_{44}y_n-A_{45}x_m \\
&K_{44}=-A_{55}-D_{11}x_m^2-2D_{16}\, x_m y_n-D_{66}y_n^2 \\
&K_{45}=K_{54}=-A_{45}-D_{16}x_m^2-(D_{12}+D_{66})\, x_m y_n-D_{26}y_n^2 \\
&K_{55}=-A_{44}-D_{66}x_m^2-2D_{26}\, x_m y_n-D_{22}y_n^2 \\
&M_{ij}=M_{ji} M_{11}=M_{22}=M_{33}=-I_1 M_{44}=M_{55}=-I_3
\end{aligned} \tag{15}$$

4. NUMERICAL SOLUTIONS AND DISCUSSIONS

In this study, free vibration analyses of symmetrically laminated composite plates on elastic foundation are investigated. Navier solutions for free vibration analysis of laminated composite plates are obtained by solving the eigenvalue equations. The plate, in hand, has a quadrangle planform where the ratio of plan-form dimensions varies from 1 to 4 (a/b=1, 2, 4). Effect of plate thickness ratio that ratio of plate width to plate thickness, a/h=100, 50, 20, 10 and 5, has been examined. In the analysis material properties are assumed to be $E_1/E_2=40$, $G_{12}/E_2=G_{13}/E_2=0.6$, $G_{23}/E_2=0.5$, $\upsilon_{12}=0.25$.

A computer program has been prepared using Mathematica program separately for the analytical solution of free vibration analysis of laminated composite plates resting on an elastic foundation. Comparisons are made with available solutions in literature. Then additional examples are solved to search the effect of lamination orientations, fiber orientations and foundation stiffness on the free vibration of laminated plates resting on elastic foundation. The results have been compared in tables and graphs.

In analysis, following parameter are used for non-dimensional free vibration frequency, non-dimensional linear Winkler foundation parameter and non-dimensional Pasternak foundation parameter as;

$$\Omega = \omega \frac{a^2}{h}\sqrt{\frac{\rho}{E_2}}, \; k_o = \frac{K_o a^4}{E_2 h^3}, \; k_1 = \frac{K_1 a^2}{E_2 h^3} \quad (16)$$

Table 1. Non-dimensional fundamental frequency parameters of antisymmetric square plate for various values of orthotropy ratio (a/b=1 and a/h=5)

	Method	**E_1/E_2** 3	10	20	30	40
$(0/90)_1$	Noor [1973]	6.2578	6.9845	7.6883	8.1763	8.5625
	Reddy [1984]	6.2169	6.9887	7.8210	8.5050	9.0871
	Kant [2001]	6.1566	6.9363	7.6883	8.2570	8.7097
	Present study	6.2086	6.9393	7.7060	8.3211	8.8333
$(0/90)_2$	Noor [1973]	6.5455	8.1445	9.4055	10.1650	10.6790
	Reddy [1984]	6.5008	8.1954	9.6265	10.5340	11.1710
	Kant [2001]	6.4319	8.1010	9.4338	10.2460	10.7990
	Present study	6.5043	8.2246	9.6885	10.6198	11.2708
$(0/90)_3$	Noor [1973]	6.6100	8.3372	9.8398	10.6950	11.2720
	Reddy [1984]	6.5552	8.4041	9.9175	10.8540	11.5000
	Kant [2001]	6.4873	8.4143	9.8012	10.6850	11.2830
	Present study	6.5569	8.4183	9.9427	10.8828	11.5264

Table 2. Non-dimensional fundamental frequency parameters of (0/90/0) square plate for various values of a/h ratio (a/b= 1 and E_1/E_2=40)

k_0	**k_1**	**Method**	**a/h** 5	10	20	50
0	0	Hui-Shen et al. [2003]	10.263	14.702	17.483	18.689
		Present study	10.289	14.766	17.516	18.648
100	0	Hui-Shen et al. [2003]	14.244	17.753	20.132	21.152
		Present study	14.263	17.805	20.161	21.158
100	10	Hui-Shen et al. [2003]	19.879	22.596	24.536	25.390
		Present study	19.891	22.637	24.560	25.396

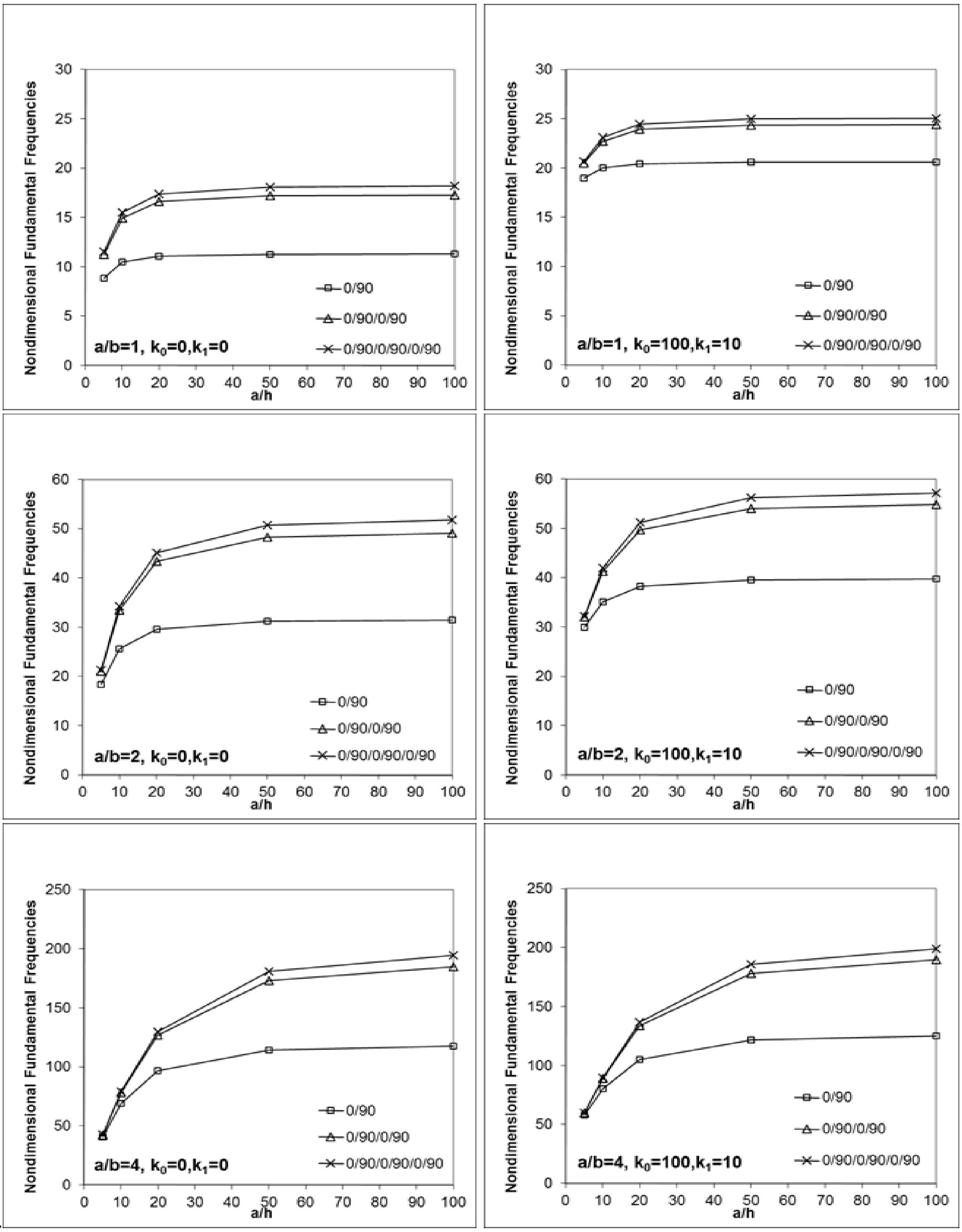

Figure 2. Effect of thickness ratio on the nondimensional frequency parameters for antisymmetric laminated composite plates on elastic foundation

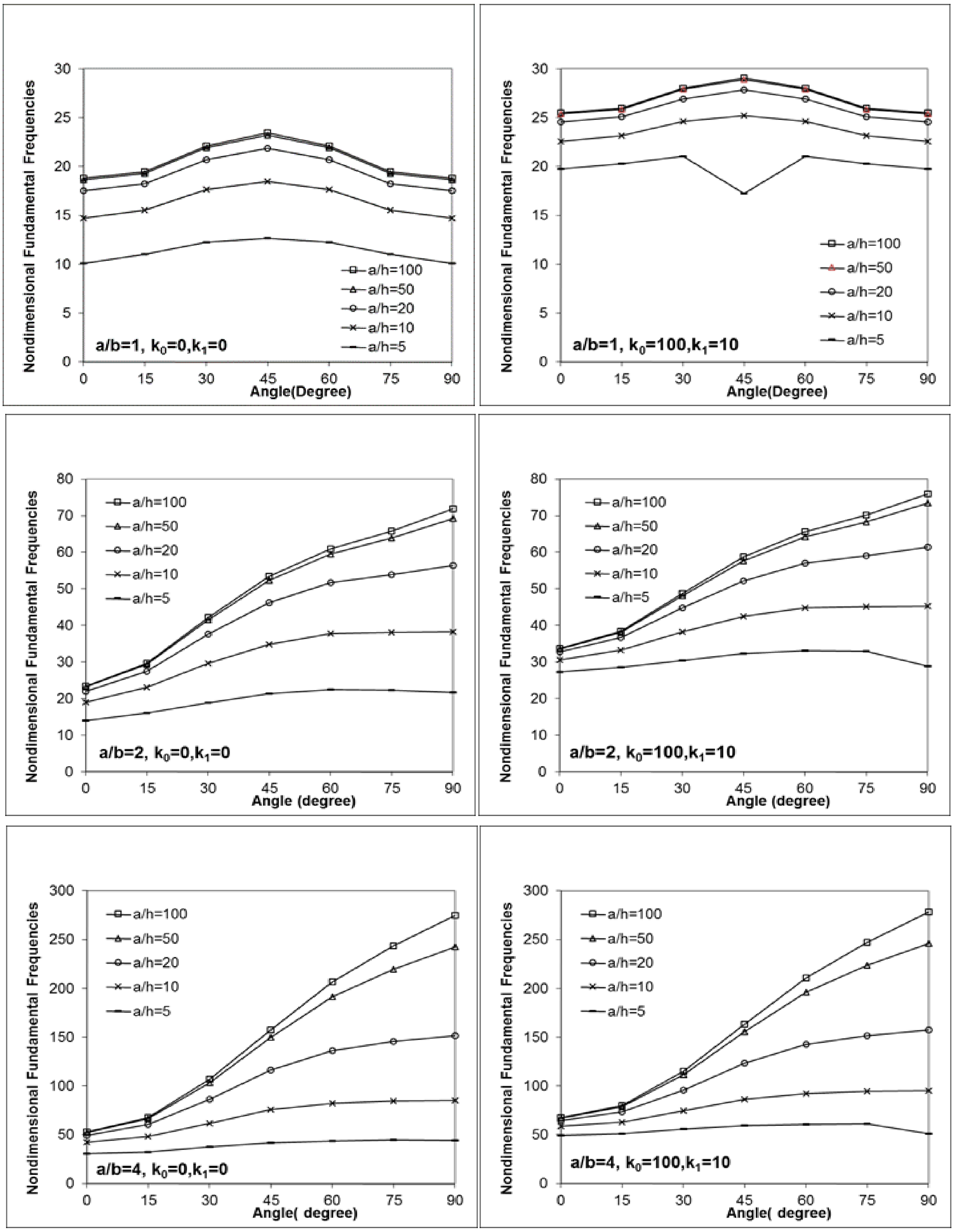

Figure 3. Effect of varying lamination angle θ on the non-dimensional frequency parameters for antisymmetric [ɵ/- ɵ/ ɵ/ -ɵ] laminated composite plates on elastic foundation

It can be seen from the Fig. 2 that effect of foundation parameter k_0 is important for thin and thick LCP. Rise in a/b ratio lead to a decreasing trend in the incremental rate of the fundamental frequency when the k_0 value changes. However, increase of Pasternak parameter (k_1) from 0 to 10 considerably affected the incremental rate of natural frequency of LCP on elastic foundation.

Fig. 3 demonstrates the influences of θ (lamination angle), a/b, a/h and foundation parameters on the natural frequency of the anti-symmetrically (ϴ/- ϴ/ ϴ/- ϴ) LCP when keeping constant the E_1/E_2 ratio at 40. Increase of θ caused to increase in dimensionless fundamental frequency regardless of the plate geometry. For all lamination angle studied caused to evident increase in the non-dimensional free vibration frequency parameters increase when foundation parameters increase. As seen from Fig. 3, when the a/b equals to 1 (square plates), results obtained for lamination angle equals to 0º, 15º are 30º are exactly same as those obtained for 90º, 75º are 60º, respectively due to symmetry.

5. CONCLUSIONS

In this study, free vibration analyses of anti-symmetrically laminated composite plates based on elastic foundation are investigated. The most important observations and results are summarized as follows:

Results showed present study and other shear deformation results for the non-dimensional frequencies are very closed.

For the cross-ply laminated composite plates, increase of foundation parameters (k_0 and k_1) increased the non-dimensional free vibration frequency parameters.

Results also showed that k_1 is more effective than that of the k_0. Results showed that a/h ratio is an effective parameter on the foundation stiffness, increase of a/h ratios increased the stiffness significantly.

For the laminated composite square plates (a/b=1), k_0 is an important parameter, however effect of k_0 is insignificant when the plate plan form turns to square from rectangle (a/b=2, or 4).

Rise in the laminate angle (ϴ) evoked the decline in displacement amplitude; however, increase in fundamental frequency irrespective of the plate geometry.

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